Exercise 111

Find $f'(x)$ if it is known that

$$\frac{d}{dx}[f(2x)] = x^2$$

Solution

Suppose that

$$\frac{d}{dx}[f(2x)] = x^2.$$

Use the chain rule on the left side.

$$f'(2x) \cdot \frac{d}{dx}(2x) = x^2$$

Evaluate the derivative.

$$f'(2x) \cdot (2) = x^2$$

Make the substitution $u = 2x$. Then $x = u/2$.

$$f'(u) \cdot (2) = \left(\frac{u}{2}\right)^2$$

$$2f'(u) = \frac{u^2}{4}$$

Divide both sides by 2.

$$f'(u) = \frac{u^2}{8}$$

Therefore, since u is just a dummy variable,

$$f'(x) = \frac{x^2}{8}.$$